Solving Linear Equations with Variables on Both Sides
Algebra 1

Often we will need to solve linear equations where the variable happens to be on both sides of the equality. The objective in solving these equations is the same as that of the simpler ones that we have seen – isolate the variable and solve for its value. The key is, as always, to manipulate the equation by doing the same thing to both of its sides.

Exercise #1: Solve the linear equation below and check your answer by using STORE on your calculator.

\[ 5x - 5 = 2x + 13 \]

As we see from this exercise, an equation truly works like a balancing scale. As long as we perform the same operation to both sides of the equation, like adding the same amount of weight to both sides of a scale, the equation remains valid - the scale remains balanced.

Exercise #2: Solve each of the following equations. Check by using STORE.

(a) \[ 7x - 21 = x + 3 \]  
(b) \[ -4 + 3x = 6x + 32 \]  
(c) \[ -2x - 18 = -4x - 6 \]

Sometimes we encounter problems where we need to combine like terms as well. It is advisable to combine like terms first.

Exercise #3: Solve each of the following equations. Check by using STORE.

(a) \[ 10 - 7x + x = 5x - 80 - 2x \]  
(b) \[ 10x - 3 - 8x = -x + 11 - 3x + 10 \]
Exercise #4: Which of the following values of \( x \) satisfies the equation \( 2x - 14 = 7x + 6 \)?

(1) \( x = -2 \)  
(2) \( x = 6 \)  
(3) \( x = 5 \)  
(4) \( x = -4 \)

It is important to continue our work with translating verbal phrases into equations. Some of these can also result in variables on both sides of the equation.

Exercise #5: Translate each of the following verbal sentences into an equation and then solve for the number described.

(a) Eight times a number is 36 more than twice the same number.

(b) If three times a number is increased by 24, the result is six times the same number.

(c) If three times a number is increased by 22, the result is 14 less than seven times the same number.

Exercise #6: A square and a rectangle are shown below with side lengths in terms of \( x \). The perimeter of the rectangle is 123 more than the perimeter of the square. Find the value of \( x \).
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Algebra 1 Homework

Skills

1. Which of the following values of \( x \) is a solution to the equation \( 18 - 4x = 6 - 16x \)?

   \[ (1) \ x = 1 \hspace{1cm} (3) \ x = -3 \]

   \[ (2) \ x = -1 \hspace{1cm} (4) \ x = 7 \]

2. Which of the following values of \( x \) is a solution to the equation \( 5x - 11 = 2x - 3 \)?

   \[ (1) \ x = 5 \hspace{1cm} (3) \ x = \frac{8}{3} \]

   \[ (2) \ x = \frac{2}{3} \hspace{1cm} (4) \ x = \frac{11}{2} \]

3. Which of the following values of \( x \) is a solution to the equation \( \frac{2}{3}x + 13 = \frac{7}{3}x - 22 \)?

   \[ (1) \ x = \frac{5}{3} \hspace{1cm} (3) \ x = 21 \]

   \[ (2) \ x = -\frac{10}{3} \hspace{1cm} (4) \ x = -15 \]

4. Solve each of the following linear equations. Check your answers using \text{STORE} on your calculator.

   (a) \( 3x + 5 = 2x - 7 \)

   (b) \( x - 2 = 3x - 1 \)

   (c) \( 3x + 4x - 7 = 8 - 3x + 11 \)

   (d) \( 5x + 6 + 2x - 1 = 10x + 3 \)

   (e) \( 2 + 7x + 8 = x - 14 \)

   (f) \( 6x - 10 - x - 2 = 9x + 2 \)
5. Translate each of the following sentences into an equation and then solve for the number described.

(a) Ten times a number increased by six is four times the same number increased by 24.

(b) Eight times a number increased by 12 is eight less than three times the same number.

(c) Nine more than two-thirds of a number is one more than the same number.

Applications

6. An isosceles triangle and a square are shown below with their side lengths given in terms of \( x \). If the perimeter of the triangle is 27 more than the perimeter of the square, find the value of \( x \).

Reasoning

7. Consider the linear equation \( 5x + 11 - 2x + 6 = 3x + 17 \).

(a) For each of the following, determine whether the given value of \( x \) satisfies the equation. Use STORE on your calculator.

\[
x = 5 \\
x = -3 \\
x = \frac{5}{3} \\
x = -\frac{3}{2}
\]

(b) Simplify the left hand side of the equation by combining like terms.

(c) What can you say about the solution set of this equation considering both parts (a) and (b)?